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COMPARATIVE FORECASTING OF MOROCCAN DIRHAM EXCHANGE RATES: ARMA VS. MARKOV SWITCHING AUTOREGRESSIVE (MS-AR) MODELS

PRÉVISION COMPARATIVE DES TAUX DE CHANGE DU DIRHAM MAROCAIN : MODÈLES ARMA VS. MARKOV SWITCHING AUTOREGRESSIVE (MS-AR)

Adil MOGHAR

Professor at National school of management ENCG - Tanger - Abdelmalek Essaâdi University,

Email: a.moghar@uae.ac.ma

Almahdi KORAICH

Professor at National school of management ENCG- Tanger - Abdelmalek Essaâdi University,

Email: a.koraich@uae.ac.ma

ABSTRACT

This paper has modelled and predicted the exchange rate between the Moroccan Dirham, the EURO and the US Dollar, for this reason two types of models have been used (the MS-AR and the ARMA model), the results of our study show that the MS-AR model outperformed the ARMA in modeling the two real exchange rates EUR/MAD and USD/MAD. The BIC information criterion shows that the MS-AR model beats the ARMA model based on random walk, while the forecasts of both models show that the ARMA has more accurate forecasts.

Keywords: Time series modeling; emerging economy; Markov regime switching; real exchange rate; econometric forecasting; Meese and Rogoff puzzle.

RÉSUMÉ

Dans la présente étude, nous avons procédé à la modélisation et à la prédiction du taux de change entre le Dirham marocain, l'Euro et le Dollar américain. À cet effet, deux modèles distincts ont été employés : le modèle à changement de régime markovien auto-régressif (MS-AR) et le modèle auto-régressif à moyenne mobile (ARMA). Les conclusions tirées de cette recherche indiquent une supériorité notable du modèle MS-AR sur le modèle ARMA dans la représentation fidèle des taux de change réels entre l'Euro et le Dirham marocain (EUR/MAD) ainsi qu'entre le Dollar américain et le Dirham marocain (USD/MAD). Selon le critère d'information bayésien (BIC), le modèle MS-AR se distingue nettement du modèle ARMA, ce dernier étant basé sur une hypothèse de marche aléatoire. Néanmoins, il convient de noter que, bien que le modèle MS-AR se soit avéré plus efficace dans la modélisation, les prévisions générées par le modèle ARMA se révèlent être d'une plus grande précision.

Mots clés : Modélisation des séries temporelles; Économie émergente; Changement de régime de Markov; Taux de change réel; Prévision économétrique; Énigme de Meese et Rogoff.

1. INTRODUCTION

The exchange rate expressed as the quantity of money needed to acquire one unit of a different currency is considered to be the cornerstone in international economic and financial relations, it is well recognized that it represents a decisive link between the internal economy of a country and the international economy, as well as being of paramount importance in determining macroeconomic stability and in providing incentives to engage in international trade. The exchange rate is unquestionably an important macroeconomic variable. For a small open economy, exchange rate adjustment helps smoothing out shocks in terms of trade. In a less open economy, it promotes the adjustment of relative prices between the tradable and non-tradable goods sectors. Empirically, the exchange rate remains the pet peeve of researchers. The empirical literature shows that it is very difficult to explain and predict its fluctuations. Fundamental questions recurrently arise, particularly concerning the ways in which the external value of money is determined and their theoretical underpinnings. The first difficulty that arises when dealing with exchange rate problems is in fact the great variety contained in the concept of the exchange rate itself. There is indeed a very wide variety of exchange rates, which is why we think it is useful to specify the type of exchange rate. This work is an attempt to predict the long-term behaviour of the dirham exchange rate. To do so, we consider two exchange rates: the real EUR/MAD and the real USD/MAD. The study of the behaviour of these rates is of crucial importance for economic stability in which the exchange rate remains a major factor in investment decisions. Is it possible to build an econometric model which explains and forecasts the behaviour of bilateral exchange rates between the Moroccan dirham, dollar and euro, without going through the fundamentals of the Moroccan economy? Our interest in this issue is inspired by the considerable lack of reliable empirical studies on the subject, and by an old and ever-renewed question, namely the forecasting of the exchange rate. We will structure our work according to the following plan. We will start with some empirical work that has been done on exchange rate modelling and forecasting. Next, we will present econometric models that specifically include regime-switching models. Finally, the empirical part of our work will deal with two real exchange rates USD/MAD and EUR/MAD: we will present their evolutions and build predictive models.

2. LITERATURE REVIEW

The study of a time series makes it possible to analyse, describe and explain a phenomenon over time and to draw consequences for decision-making. One of the main objectives of modelling is forecasting, which consists of predicting the future values of the series from its observed values. In this section we will present some non-linear models such as TAR, STAR, SETAR and MS-AR models.

2.1.FORECASTING THE LEVEL OF EXCHANGE RATES

As the literature on the use of non-linear models, and more particularly Markov regime-switching models, is constantly evolving, the present axis is a synthetic presentation of the most relevant work. Bergman and Hansson (2005) suggested that the real exchange rate can be modelled using a stationary two-state Markov regime-switching model. Using log-transformed quarterly data (between 1973 and 1997) associated with the spot exchange rate (in units of domestic currency per US dollar) and the consumer price index for six major industrialized countries: The

United Kingdom, France, Germany, Switzerland, Canada, Japan and the United States of America, the authors compare the out-of-sample predictive quality of different empirical models on the basis of their mean-squared errors (MSEs). They found that a simple model where only the constant varies across regimes has better predictive quality than both random walk and Markov regime-switching random walk models. Predictive quality tests also showed that predictions from their basic model are significantly better than those from a 1- or 2-state random walk model for all currencies except the Japanese yen. This is surprising since exchange rate models are often outperformed by random walk models. Simulation exercises have confirmed this view. As with previous results by Perron (1990) who examined the power of unit root tests in the presence of a single break in the constant, they found that the ADF test is unable to distinguish between a random walk and multiple changes in the constant of Markov regime-switching models. Therefore, it is evident that ADF tests on real post-Bretton Woods data generally fail to reject the null hypothesis of unit root in real exchange rate data. Nikolsko-Rzhevskyy and Prodan (2012) argued that the relative success of fundamental macroeconomic models in forecasting exchange rates is partly due to the drift term, and also to the fact that a simple two-state Markov Switching Random Walk (MS-RW) model with drift is a good representation of the nominal exchange rate. The authors used a version of Engel and Hamilton's (1990) model and exchange rates for 12 OECD countries against the US dollar from March 1973 to January 2008. In order to assess the out-of-sample performance of the Markov regime-switching model relative to the random-walk model, the analysis applied the inference procedure proposed by Clark and West (2006, 2007) to test the null predictive power of two nested models. The authors provided strong evidence for both short- and long-term predictability. The results show that the model significantly outperforms the random walk for nine of the 12 countries at the one-month horizon. In addition, they found strong evidence of predictability at the long horizon (more than one month), up to the three-month for seven countries and up to the 12-month for three countries. A common feature for all countries is that the evidence of predictability decreases as the forecast horizon increases. The authors have addressed the criticisms of Faust et al (2003) and Rogoff and Stavrakeva (2008) and have verified the robustness of their results at short horizons. They remain consistent when the Clark and West or Clark and McCracken statistics are used. The significance of the results decreases when the analysis uses Diebold and Mariano's U-tests, but is still superior to many previous studies. The results also remain robust for the different forecast windows and sampling end points. The data include monthly nominal exchange rates for 12 currencies: Japanese yen, Swiss franc, Australian dollar, Canadian dollar, British pound, Swedish krona, Danish krone, Deutsche Mark, French franc, Italian lira, Dutch guilder and Portuguese escudo. The exchange rate is defined as the price in US dollars per unit of foreign currency. Data starts in March 1973 and ends in January 2008 for most countries. However, the exchange rates for the five countries of the euro zone are only examined in December 1998, before the introduction of the euro.

3. METHODOLOGY

The study of a time series makes it possible to analyse, describe and explain a phenomenon over time and to draw consequences for decision-making. One of the main objectives of modelling is forecasting, which consists of predicting the future values of the series from its observed values. In this section we will present some non-linear models such as TAR, STAR, SETAR and MS-AR models.

3.1.NOTION OF STATIONARITY

The notion of a stationary series is indispensable for the further analysis of the series. A stationary series Xt is a series whose properties are unchanged by a change in time origin, i.e.:

a. The expectation of Xt is constant over time: one can think of "expectation" as mathematical expectation, but one must keep in mind that it is the expected value for observation at time t.

b. For any fixed h, the covariance between Xt and Xt+h is invariant in time. This implies, among other things, that the series is stable in dispersion. This notion of dispersion of the series is very important for the continuation. We can make the series scatterplot with the values of the series shifted by one shift h.

Stationarity implies that if you take a certain number of points at the beginning or at the end of the series you must always find the same point cloud.

3.2.REGIME-SWITCHING MODELS

A time-series model links a series of observations $\{y_t\}_{t=1}^{T}$ to a series of shocks $\{y_t\}_{t=1}^{T}$ uncorrelated, zero mean and variance σ_{ε}^{2}. in a linear configuration, Wold's (1954) representation theorem states that any stationary process $\{y_t\}_{t=1}^{T}$ can be written as an infinite moving average of past and present shocks:

$$y_t = \mu + \sum_{i=0}^{\infty} \emptyset_i \varepsilon_{t-i}, \forall t = 1,2,\dots,T$$

With μ a constant and $\emptyset_i$ real coefficients such as $\emptyset_0 = 1$. In the case of a non-linear representation, the series $\{y_t\}_{t=1}^{T}$ is related to the history of the shocks $\{\varepsilon_t\}_{t=1}^{T}$ through a non-linear function f (-)

$$y_t = f(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots), \forall t = 1,2,\dots,T$$

Since this writing is very general, Campbell et al (1996) propose a more restrictive framework to describe a non-linear process:

$$y_t = g(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots), \varepsilon_t \sqrt{h(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots)} \forall t = 1,2,\dots,T$$

Where the function g (-) is the conditional expectation of yt, and h (-) its conditional variance. Therefore, depending on whether it is the function g (-) or h (-) that is non-linear, a process can be non-linear in mean and/or variance. A model that is non-linear in mean is used to model the returns of an asset or a portfolio of assets, while a model that is non-linear in variance is particularly suitable for risk modelling.

3.3.MODELS WITH OBSERVABLE TRANSITION VARIABLE

Introduced by Tong (1978), Tong and Lim (1980) and Tong (1993), the TAR (Threshold Auto-Regression) model is a piecewise linear model, each determined by the position of the transition variable xt with respect to a given threshold c. formally, a TAR model with 2 regimes of orders p1 and p2, noted TAR (2 ; p1 ; p2), is written :

$$y_t = \{\emptyset_0^1 + \emptyset_1^1 y_{t-1} + \emptyset_2^1 y_{t-2} + \cdots + \emptyset_{p_1}^1 y_{t-p_1} + \varepsilon_t^1 si x_t \leq c \emptyset_0^2 + \emptyset_1^2 y_{t-1} + \emptyset_2^2 y_{t-2} + \cdots + \emptyset_{p_1}^2 y_{t-p_1} + \varepsilon_t^2 si x_t \leq c, \forall t = 1,2,\dots,T$$

With $\emptyset_0^j$ the regime constant j, $\{\emptyset_1^j, \emptyset_2^j, \dots, \emptyset_{pj}^j\}$ its autoregressive coefficients and ε_t^j a white noise of variance σ_j^2(j $\in \{1,2\}$). The transition variable can be the variable itself delayed by d periods, we then obtain the SETAR (Self Exciting Threshold Auto-Regression) model, noted SETAR (2; p1; p2).

$$y_t = \{\emptyset_0^1 + \emptyset_1^1 y_{t-1} + \emptyset_2^1 y_{t-2} + \cdots + \emptyset_{p_1}^1 y_{t-p_1} + \varepsilon_t^1 si y_{t-d} \leq c \emptyset_0^2 + \emptyset_1^2 y_{t-1} + \emptyset_2^2 y_{t-2} + \cdots + \emptyset_{p_1}^2 y_{t-p_1} + \varepsilon_t^2 si y_{t-d} > c, \forall t = 1,2,\dots,T$$

In general, threshold models can be written more compactly as follows:

$$y_t = (\emptyset_0^1 + \emptyset_1^1 y_{t-1} + \emptyset_2^1 y_{t-2} + \cdots + \emptyset_{p_1}^1 y_{t-p_1} + \varepsilon_t^1) l(q_t \leq c)$$

$$+(\emptyset_0^2 + \emptyset_1^2 y_{t-1} + \emptyset_2^2 y_{t-2} + \cdots + \emptyset_{p_1}^2 y_{t-p_1} + \varepsilon_t^2) l(q_t > c), \forall t = 1,2, \ldots, T$$

With l(-) the indicator function which is 1 if its argument is true and 0 if not. The transition variable is qt = xt in the case of a TAR model and qt = yt-d in the case of a SETAR model. In these models, the transition from one regime to another is abrupt because of the discrete nature of the indicator function. It is possible to consider a smooth transition function G (-), taking a continuum of values from 0 to 1 as qt increases, and this gives the STAR (Smooth Threshold Auto-Regression) model, denoted STAR (2; p1; p2):

$$y_t = (\emptyset_0^1 + \emptyset_1^1 y_{t-1} + \emptyset_2^1 y_{t-2} + \cdots + \emptyset_{p_1}^1 y_{t-p_1} + \varepsilon_t^1) G(q_t; \gamma, c)$$

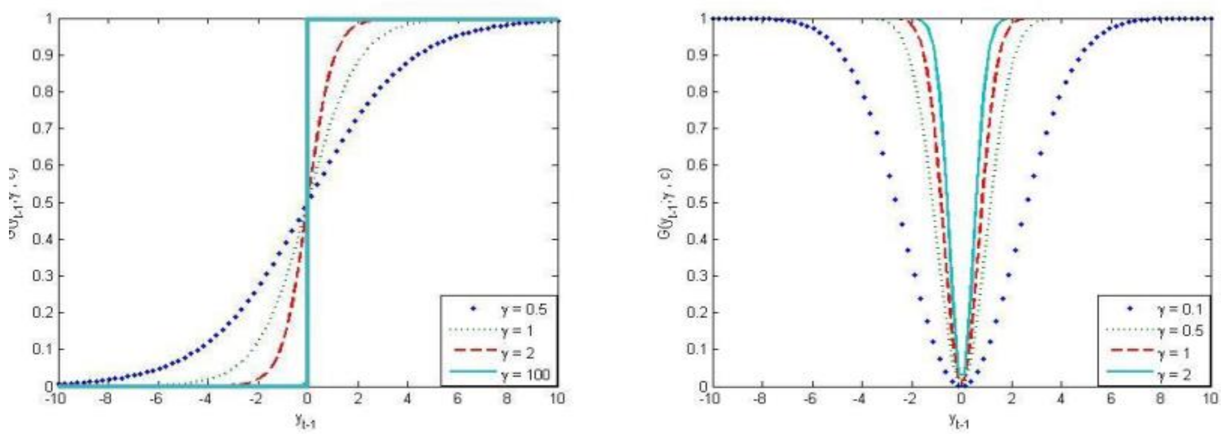
$$+\left(\emptyset_0^2 + \emptyset_1^2 y_{t-1} + \emptyset_2^2 y_{t-2} + \cdots + \emptyset_{p_1}^2 y_{t-p_1} + \varepsilon_t^2\right)(1 - G(q_t; \gamma, c)), \forall t = 1,2, \ldots, T$$

Where γ is a smoothing parameter of the transition function G (-). The idea of a smooth transition was introduced by Chan and Tong (1986) and later popularised by Granger and Teräsvirta (1993) and Teräsvirta (1994). A first possibility of a smooth transition function is offered by the logistic function, which leads to the LSTAR (Logistic Smooth Auto-Regression) model:

$$G(q_t; \gamma, c) = \frac{1}{1 + exp[-\gamma(q_t - c)]}$$

Note that if the smoothing parameter γ is large enough, the logistic function instantly changes from 0 to 1 as soon as the quantity qt - c changes sign, thus approaching the indicator function. Therefore, the LSTAR model is a good approximation to the TAR and SETAR models. On the other hand, if γ tends towards 0, then the logistic function stabilises around 0.5 and the LSTAR model is reduced to a simple linear AR model. Another possibility of transition function is given by the exponential function. In this case, the STAR model becomes ESTA R (Exponential Smooth Threshold Auto-Regression):

$$G(q_t; \gamma, c) = 1 - exp[-\gamma(q_t - c)^2]$$



(a) Fonction logistique (b) Fonction exponentielle

Fig -1: Transition functions

In contrast to the previous configuration, the effect of the sign of qt - c is neutralised by the quadratic form. As a result, the more γ is important, the longer the ESTAR model remains on the same regime and only leaves it when this parameter weakens or when the transition variable qt approaches the threshold c, but in no case does it reduce to a linear specification of type AR.

3.4.MODELS WITH UNOBSERVABLE TRANSITION VARIABLES

The transition variable qt is not necessarily always observable. It can be hidden and follow a random process in which case we only have probabilities about the values it can take. Consequently, the regime in which the explained variable lies is never known with certainty. The best-known model in this class is the Markov Switching Auto Regression (MS-AR) model introduced by Hamilton (1989) to model US GDP. In this model, the switching variable follows a Markov chain of order 1 with a number of states equal to the number of regimes. The MS (2)-AR (p1; p2) model, for example, is written as:

$$y_t = \{\emptyset_0^1 + \emptyset_1^1 y_{t-1} + \emptyset_2^1 y_{t-2} + \cdots + \emptyset_{p_1}^1 y_{t-p_1} + \varepsilon_t^1 siR_t = 1\emptyset_0^2 + \emptyset_1^2 y_{t-1} + \emptyset_2^2 y_{t-2} + \cdots + \emptyset_{p_1}^2 y_{t-p_1} + \varepsilon_t^2 siR_t = 2, \forall t = 1,2,\ldots,T$$

For the model to be fully specified, it is necessary at the same time to characterise the transition matrix P associated with the Markov chain:

$$\left(\frac{p_{11}1 - p_{22}}{1 - p_{11}p_{22}}\right)$$

In which the elements pij represent the probability of moving from a regime Rt-1 = i to a regime Rt=j

$$p_{ij} = P(S_t = j | S_{t-1} = i),, \forall t = 1,2,\ldots,T, j \in \{1,2\}$$

4. DATA END RESULTS

The objective of this section is to find the best model for forecasting. First we will present the data their frequencies and period. Then we will move on to the main descriptive statistics. Then we will move on to modeling the two exchange rate series using linear and non-linear models, as well as applying forecasting methods.

4.1.MODELS WITH UNOBSERVABLE TRANSITION VARIABLES

This section presents all the variables, their frequencies, descriptive statistics as well as the evolution graphs of the two main variables for our study, corresponding to the real EUR/MAD and USD/MAD exchange rates. The data available are composed of two series of variables, the real USD/MAD exchange rate and the real EUR/MAD exchange rate. The data come from Banque Al Maghreb (BAM). Our sample is composed of five variables for a period from January, 1999 to May, 2017. The data correspond to the two real exchange rates EUR/MAD, USD/MAD. The data have a monthly frequency with a total number of 221 observations.

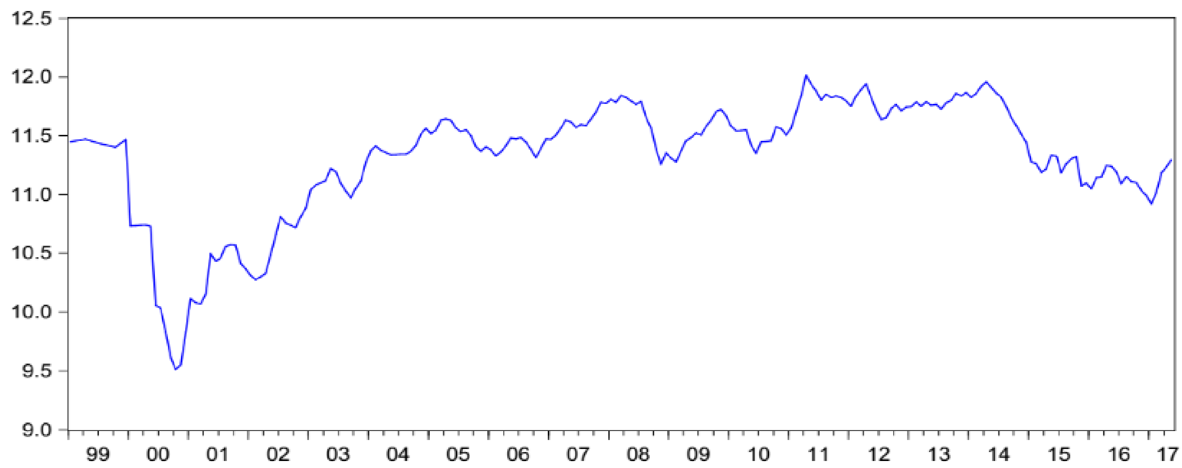


Fig -2: Real EUR/MAD exchange rate

This curve represents the closing rates of the real exchange rate of the US dollar against the Moroccan dirham. This curve contains 221 monthly frequency observations for the period from January 1999 to May 2017.

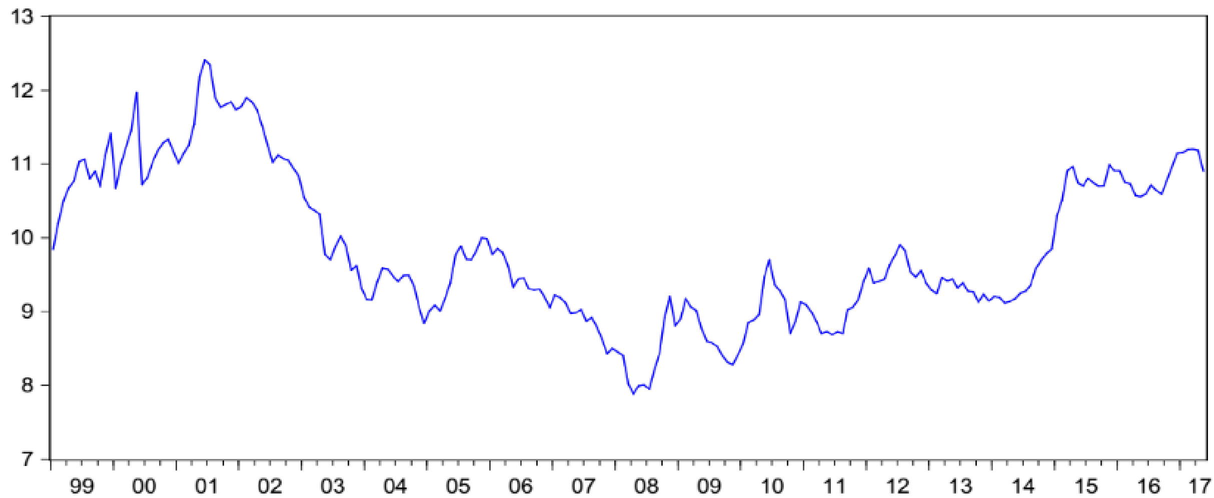


Fig -3: Real USD/MAD exchange rate

This curve represents the evolution of the real exchange rate of the euro against the Moroccan dirham, this curve contains 221 monthly frequency observations for the period from January 1999 to May 2017. Both curves visually show a negative correlation relationship, when the values of the real USD/MAD exchange rate increase, the values of EUR/MAD decrease.

Table -1: Descriptive statistics for the real exchange rate variables EUR/MAD and USD/MAD

	USD/MAD	EUR/MAD
Average	9.222937	10.95300
Median	8.885880	11.06737
Maximum	11.90309	11.47288
Minimum	7.256963	9.507155

Standard deviation	1.146570	0.401148
Skewness	0.595362	-1.486392
Kurtosis	2.317311	5.151879
Jarque-Bera	17.34747	124.0181
Probability	0.000171	0.000000
Comments	221	221

The real EUR/MAD exchange rate is less volatile with a standard deviation of 0.401148 against a standard deviation of 1.146570 for the USD/MAD exchange rate. The EUR/MAD real exchange rate series shows an excess of kurtosis 3 < 5.151879, while there are thicker than normal tails. In contrast the real USD/MAD exchange rate series presents a lower-than-normal kurtosis 2.317311< 3 The Skewness Asymmetry Coefficient showed that the real USD/MAD exchange rate series has a positive skewness to the right, in contrast to the real EUR/MAD exchange rate series which has a negative skewness to the left. These last two coefficients confirm the non-normality of the distributions of the two real exchange rate series. Moreover, we accept the hypothesis of normality if JB < 5.99 at the 5% threshold), otherwise we reject the hypothesis that confirms our result.

4.2.REAL EXCHANGE RATE STATIONARITY TEST

Time series modelling requires that time series be stationary. In other words, the series has no trends, cycles or seasonality. This notion of stationarity represents a crucial point in time series econometrics, where the estimation of non-stationary series leads to spurious or illusory regressions. To avoid these spurious estimates, econometricians use the stationarity of time series. A time series is said to be stationary if its mean, variance and covariance are constant (unaffected by time). These conditions stipulate that the expectation, variance and covariance are constant over time. Note that the variance must be finite. If one of these conditions is not verified, we speak of non-stationarity. To test the non-stationarity of the series, the Augmented Dickey-Fuller test is used. The test consists of testing for the existence of a deterministic trend in the series. In other words, testing the null hypothesis which stipulates that the coefficient associated with the trend is significantly equal to zero.

Table -2: Stationarity test of the real EUR/MAD and USD/MAD exchange rate

		EUR/MAD			**USD/MAD**		
		Test statistics	**Critical value**	**Probability**	**Test statistics**	**Critical value**	**Probability**
ADF	**Trend intercept**	-2,306837	-4,001	0,43	-1,4445	-4,00051	0,8452
	Intercept	-1,905921	-3,46	0,33	-1,5934	-3,46017	0,4842
	No	-0,162522	-2,576	0,63	0,0105	-2,57556	0,685

Both real exchange rates are characterized by non-stationarity. The ADF test values for the models show the existence of a unit root.

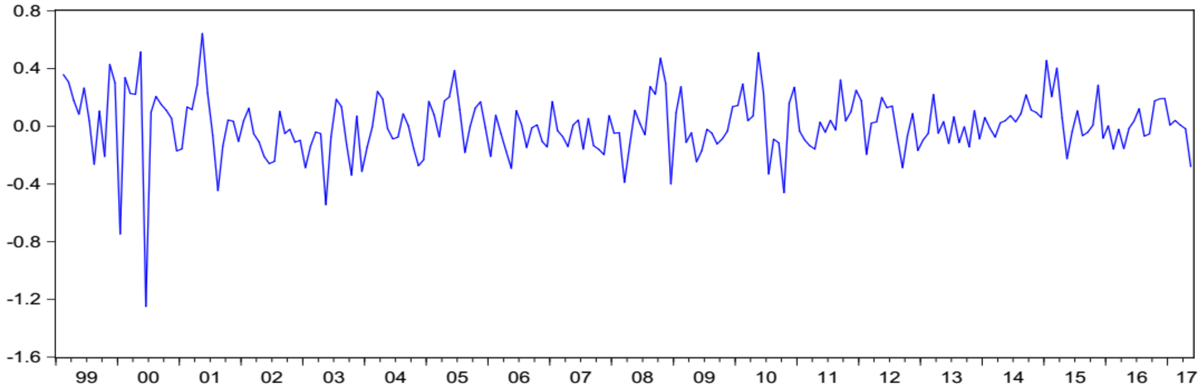


Fig -4: Evolution of the real USD/MAD exchange rate variation

This curve represents the first difference in the real exchange rate of the US dollar against the Moroccan dirham, the series has a trend of zero average as well as a finite variation.

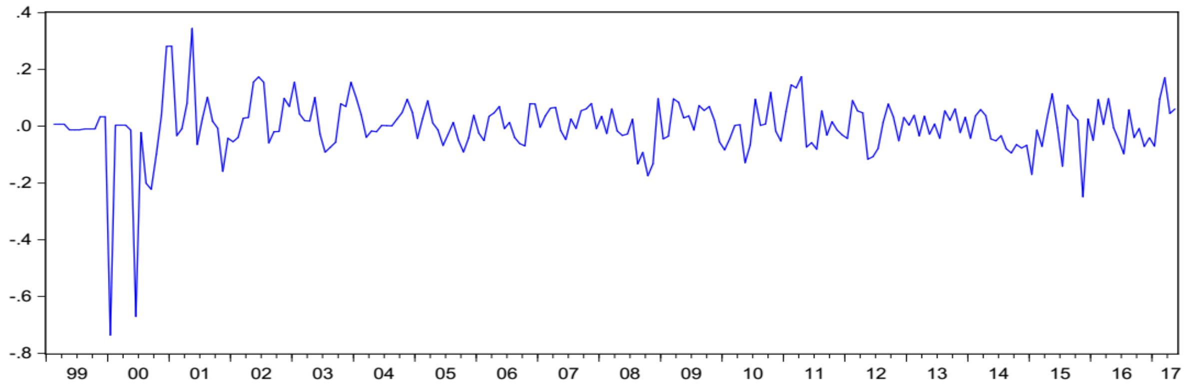


Fig -5: The evolution of the variation in the real EUR/MAD exchange rate

This curve represents the first difference in the real exchange rate of the euro against the Moroccan dirham, the series has a tendency to zero average as well as a finite variation.

Table -3: Stationarity test of the 1st difference of the real exchange rates USD/MAD EUR/MAD

		1st difference EUR/MAD			1st difference USD/MAD		
		Test statistics	**Critical value**	**Probability**	**Test statistics**	**Critical value**	**Probability**
ADF	**Trend intercept**	-11,965	-4,00051	0	-13,0801	-4,00051	0
	Intercept	-11,9919	-3,46017	0	-13,0781	-3,46017	0
	No	-12,0191	-2,57556	0	-13,1077	-2,57556	0

The ADF test shows that the two series of real exchange rates EUR/MAD, USD/MAD are stationary for all models, the t-statistic values are well below the critical values confirming the stationarity of the series.

4.3.MODELING

In order to model the behavior of a time series, the stationarity of level series must be tested. If the stationarity test shows the existence of a unit root, subtract the trend if the trend component is deterministic, differentiate the series if the series represents a random walk with or without drift. In what will follow, we will proceed to the varied unitary modelling of a time series through the ARMA models testing the existence of regime change in the series, then the modelling by the MS-AR Markov Switching Autoregressive Model regression model. In order to carry out the modelling and forecasting for each series, two periods will be taken, the first one from 01/1999 to 05/2015 for the modelling and one period from 06/2015 to 05/2017 for the forecasting.

Table -4: ARMA model parameters for the two series of 1st difference in real exchange rate USD/MAD and EUR/MAD

	1st difference EUR/MAD		**1st difference USD/MAD**	
	Coefficient	Probability	Coefficient	Probability
C	-0,000594	0.9374	0,004598	0,7694
BIC	-1,633782		-0,175414	

The probability values of the constant for both series exceed 0.05 which means that the constants are neglected for both series while the BIC values are -1.633782 for 1st difference of the real EUR/MAD exchange rate and -0.175414 for 1st difference of the real USD/MAD exchange rate.

Table -5: Forecast results based on the ARMA model for the 1st difference in the real exchange rate EUR/MAD and USD/MAD

Estimation based on the ARMA model for the 1st difference EUR/MAD		**ARMA model-based estimate for the 1st difference USD/MAD**	
Root Mean Squared Error	0.087547	Root Mean Squared Error	0.123101
Mean Absolute Error	0.067380	Mean Absolute Error	0.090580

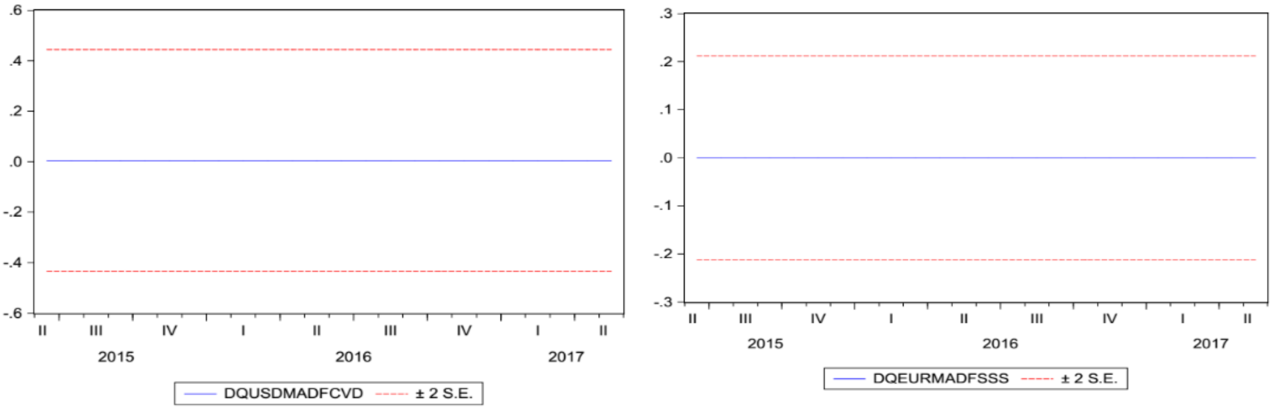


Fig -6: ARMA model-based forecast of real USD/MAD and EUR/MAD exchange rates

The forecast values are given by the two criteria RMSE and MAE their values are respectively 0.087547 and 0.067380 for estimation on the basis of the ARMA model for the 1st difference of real exchange rate EUR/MAD and 0.123101 and 0.090580 for estimation on the basis of the ARMA model for the 1st difference of real exchange rate USD/MAD.

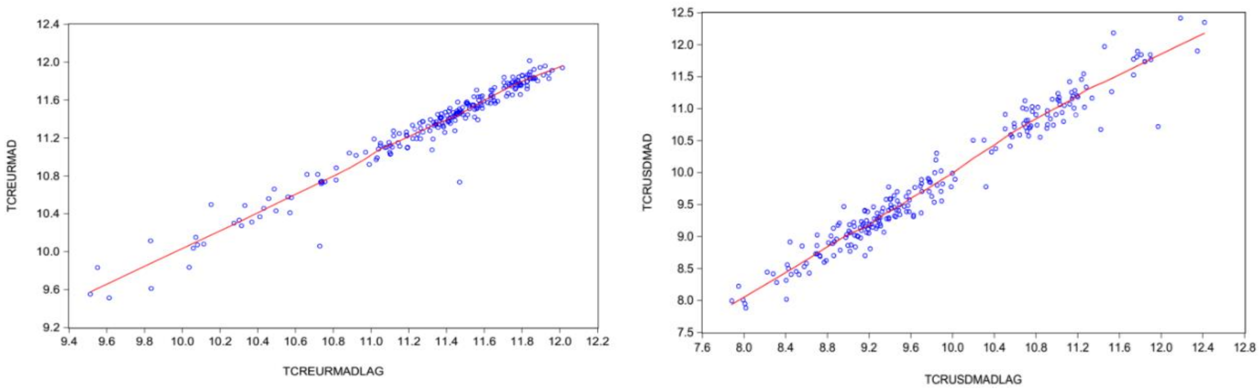


Fig -7: Scatter plot of the two real exchange rates with their first lags

The two scatter plots highlight the non-linear relationship between the two exchange rates and their respective pasts. It can be seen that the two lines obtained are not completely linear:

- For the euro, the line is broken at around 11.08
- For the dollar it is broken around 10.6

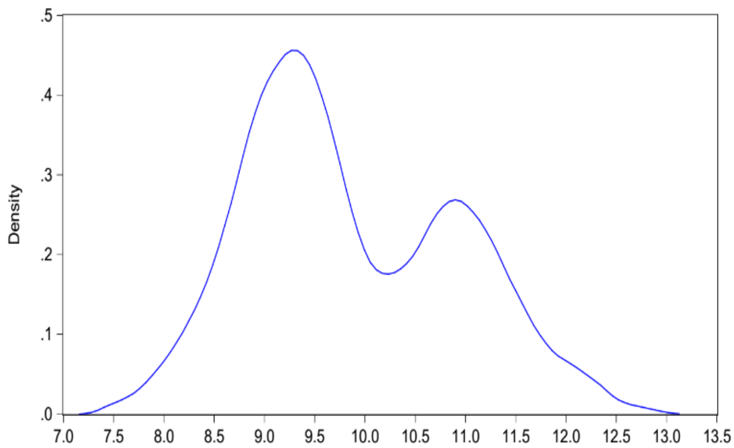


Fig -8: The density function of the real USD/MAD exchange rate

This density function of the real USD/MAD exchange rate is characterised by two different regimes.

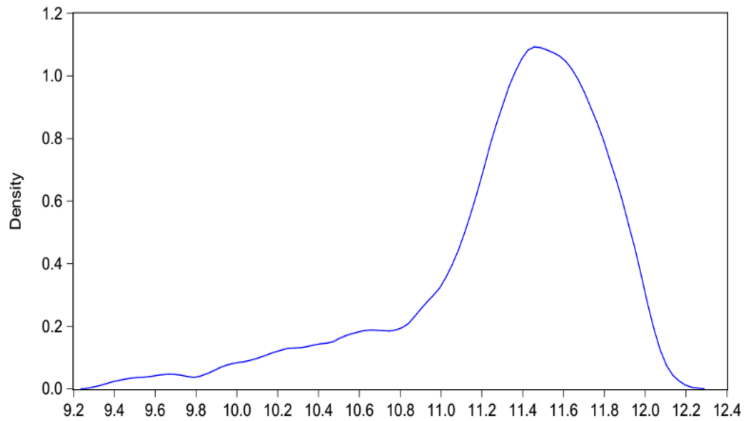


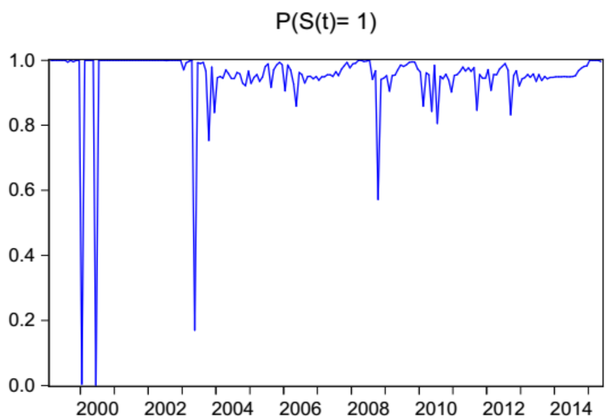
Fig -9: The density function of the real EUR/MAD exchange rate

In this density function of the real EUR/MAD exchange rate, only one regime is observed

Table -6: MS-AR model estimation parameter for the two real exchange rate variables EUR/MAD and USD/MAD

		Plans 1	**Plans 2**
EURMAD	**Alpha**	0.926220	0.973370
	BIC	-2.004090	
USDMAD	**Alpha**	0.589211	1.003598
	BIC	-0.200707	

The alpha parameters of the model are given respectively by 0.926220, 0.973370 for regimes 1 and 2 of the real EUR/MAD exchange rate and then 0.589211, 1.003598 for regimes 1 and 2 of the real USD/MAD exchange rate. The BIC criterion for the real EUR/MAD exchange rate is -2.004090 and for the real USD/MAD exchange rate is -0.200707.



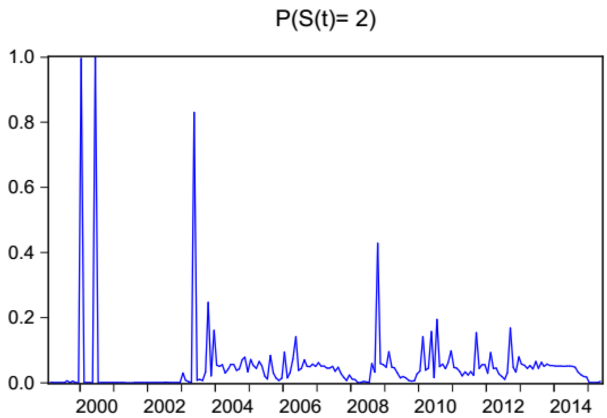


Fig -10: Probability of real USD/MAD exchange rate regime shift

In the case of the real USD/MAD exchange rate, the probability of regime change is present for almost the entire period except for the years 2001 and 2002, which are stabilized in regime 1 during this period.

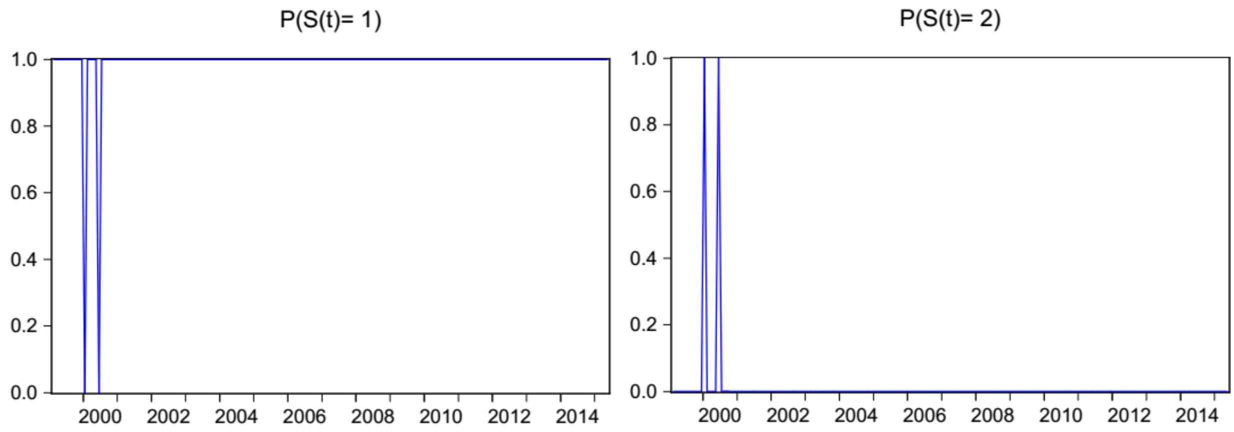


Fig -11: Probability of a change in the real EUR/MAD exchange rate regime

For the real EUR/MAD exchange rate, the probability that the regime changes from 1 to 2 is present only in the period of the years 2000, 2001, whereas for the rest the regime1 retains a probability of 100%.

Table -7: Forecast results based on MS-AR model of the real EUR/MAD and USD/MAD exchange rate

MS-AR model-based estimate for the real EUR/MAD exchange rate		**MS-AR model-based estimate for the real USD/MAD exchange rate**	
Root Mean Squared Error	0.202837	Root Mean Squared Error	0.330091
Mean Absolute Error	0.172259	Mean Absolute Error	0.234150

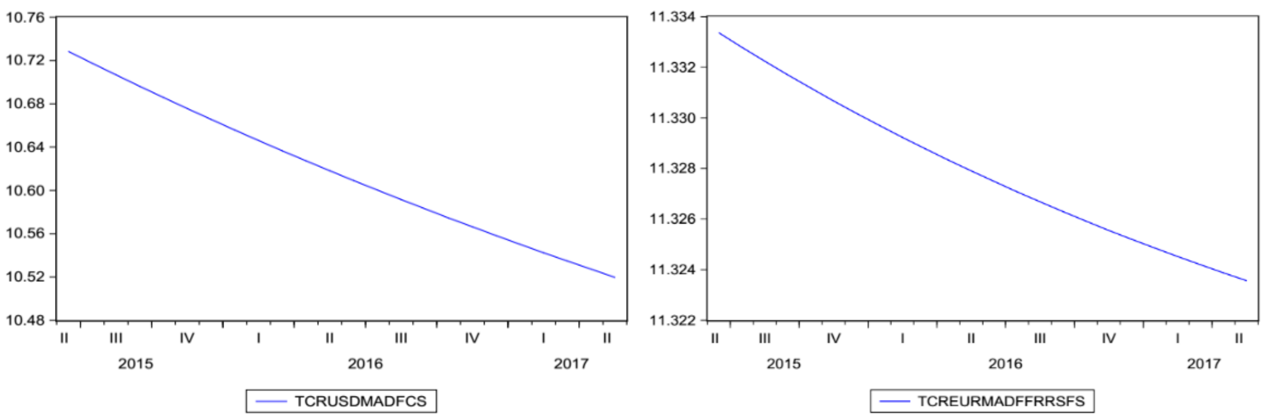


Fig -12: MS-AR model-based forecast of real exchange rates USD/MAD and EUR/MAD

The values of the forecasts are given by the two criteria RMSE and MAE their values are respectively real EUR/MAD exchange rate and 0.330091 and 0.234150 for estimation based on the MS-AR model for the real USD/MAD exchange rate.

4.4.RESULTS INTERPRETATION

This section summarises our work, first we will present the stationarity results of the two exchange rate series for the ADF test at the level and at the first difference, then we will move on to the selections of the best model according to the BIC selection criteria and according to the values of the estimation parameters RMSE (Root Mean Squared Error) and MAE (Mean Absolute Error).

Table -8: Stationarity tests of the two series of the real exchange rate in the level and at the 1st difference

		1st difference EUR/MAD			1st difference USD/MAD		
		Test statistics	Critical value	Probability	Test statistics	Critical value	Probability
ADF	**Trend intercept**	-11,965	-4,00051	0	-13,0801	-4,00051	0
	Intercept	-11,9919	-3,46017	0	-13,0781	-3,46017	0
	No	-12,0191	-2,57556	0	-13,1077	-2,57556	0
		EUR/MAD			**USD/MAD**		
	Trend intercept	-2,306837	-4,001	0,43	-1,4445	-4,00051	0,8452
	Intercept	-1,905921	-3,46	0,33	-1,5934	-3,46017	0,4842
	No	-0,162522	-2,576	0,63	0,0105	-2,57556	0,685

The results obtained show that the real EUR/MAD and USD/MAD exchange rates are not basically stationary - the ADF test probability values are above 0.05. This confirms the existence of a unit root, but after the calculation of the first difference it can be seen that both exchange rates have become stationary.

Table -9: Criteria for selecting the best model according to BIC, RMSE and MAE

EURMAD	**BIC**	-2,004090	**MS-AR model-based estimate for the real EUR/MAD exchange rate**	Root Mean Squared Error	0.202837
				Mean Absolute Error	0.172259
USDMAD		-0,200707	**MS-AR model-based estimate for the real USD/MAD exchange rate**	Root Mean Squared Error	0.330091
				Mean Absolute Error	0.234150
1st difference EUR/MAD		-1,633782	**Estimation based on the ARMA model for the 1st difference EUR/MAD**	Root Mean Squared Error	0.087547
				Mean Absolute Error	0.067380

1st difference USD/MAD		-0,175414	**Estimation based on ARMA model for the 1st difference USD/MAD**	Root Mean Squared Error	0.123101
				Mean Absolute Error	0.090580

5. CONCLUSION

In this study, we investigated the predictive performance of two econometric models — the ARMA model and the Markov Switching Autoregressive (MS-AR) model — applied to the real exchange rates of EUR/MAD and USD/MAD. The main objective was to determine which model offers better forecasting performance within the context of an emerging economy like Morocco. Our findings reveal that the MS-AR model outperforms the ARMA model in terms of goodness of fit, as indicated by the Bayesian Information Criterion (BIC). This suggests that the MS-AR model is more capable of capturing structural dynamics and nonlinear behaviors in exchange rate data. However, somewhat paradoxically, the ARMA model provides more accurate forecasts in terms of Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). This may be attributed to the model's simpler structure, which can sometimes generalize better and avoid overfitting. These dual findings highlight an essential lesson in time series econometrics: superior model fit does not necessarily imply better predictive performance. They also underscore the complexity inherent in modeling exchange rates — a domain where the Meese and Rogoff puzzle continues to challenge empirical forecasting efforts.

From a methodological standpoint, our analysis suggests that regime-switching models, although more demanding in terms of estimation and data requirements, offer a robust alternative to capturing the nonlinearities and structural shifts often observed in economic time series. This is particularly relevant for emerging markets like Morocco, where external shocks — political, monetary, or geopolitical — frequently affect currency behavior. Looking ahead, several research avenues could be pursued. Future studies could incorporate macroeconomic explanatory variables within a structural MS-VAR framework or extend the dataset beyond 2017 to assess the impact of recent monetary policy changes in Morocco. Additionally, integrating machine learning techniques — such as recurrent neural networks or hybrid models like ARIMA–LSTM — could offer a valuable comparison between classical econometric approaches and modern data science methods in exchange rate forecasting.

REFERENCES

Abounoori, E., Elmi, Z. M., & Nademi, Y. (2016). "Forecasting Tehran stock exchange volatility; Markov switching GARCH approach." Physica A: Statistical Mechanics and its Applications, 445, 264-282.

Bergman, U. M., & Hansson, J. (2005). Real exchange rates and switching regimes. Journal of International Money and Finance, 24(1), 121-138.

Clark, T. E., & West, K. D. (2006). "Using out-of-sample means squared prediction errors to test the martingale difference hypothesis." Journal of Econometrics, 135(1-2), 155-186.

Clark, T. E., & West, K. D. (2007). "Approximately normal tests for equal predictive accuracy in nested models." Journal of Econometrics, 138(1), 291-311.

Engel, C., & Hamilton, J. D. (1990). "Long swings in the dollar: Are they in the data and do markets know it?" The American Economic Review, 689-713.

Faust, J., Rogers, J. H., & Wright, J. H. (2003). "Exchange rate forecasting: the errors we've really made." Journal of International Economics, 60(1), 35-59.

Nikolsko-Rzhevskyy, A., & Prodan, R. (2012). "Markov switching and exchange rate predictability." International Journal of Forecasting, 28(2), 353-365.

Perron, P. (1990). "Testing for a unit root in a time series with a changing mean." Journal of Business & Economic Statistics, 8(2), 153-162.

Rogoff, K. S., & Stavrakeva, V. (2008). "The continuing puzzle of short horizon exchange rate forecasting (No. w14071)." National Bureau of Economic Research.